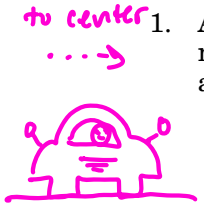
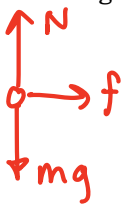
Newton's Laws – Circular Motion

For each of the problems, include a correctly labeled free-body diagram. Identifying all the forces involved in these problems is the key to doing them correctly. Remember that "centripetal force" is just a fast way of saying that the net force is causing a centripetal acceleration.

1. A 1750 kg car is traveling around in circles in a flat level parking lot. It is going a constant 7.5 m/s with a constant radius of 22 m.



a. What is the magnitude of the net force on the car?



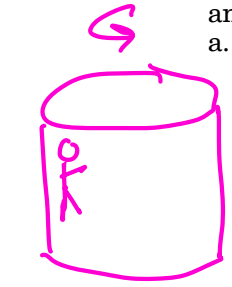
$$\Sigma F = ma = m\frac{v^2}{r}$$

$$\Sigma F = \frac{(1750)(7.5)^2}{(22)} = 4474\ N$$

b. Where does the centripetal force come from?

friction between the tires & the road. If there is no friction, car will just continue in a straight @ constant speed. (Think about icy roads.)

2. A 65 kg person is riding the Turkish Twist at Canobie Lake Park. It has a radius of 5 meters and is spinning at a constant 20 rpm when the floor drops, leaving the person stuck to the wall.



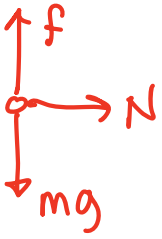
a. What is the magnitude of the net force on the person?

1st convert "rpm" to "m/s" – just dimensional analysis

$$\left(\frac{20\ rev}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{2\pi(5)\ m}{1\ rev}\right) = 10.47\ m/s$$

$$\Sigma F = ma = m\frac{v^2}{r} = \frac{(65)(10.47)^2}{5} = 1426\ N$$

b. Where does the centripetal force come from?



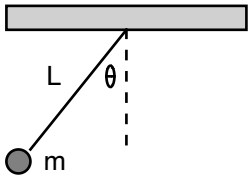
The normal force of the wall pushing the rider to the center of the circle. (Because ride is spinning, rider "wants" to continue in straight line but the wall pushes them in.)

c. Why does the rider not slide down?

friction! Gravity pulls rider down & friction between wall & rider prevents that (as long as "μN" is large enough. Don't wear a silk skirt on the Turkish Twist!

Newton's Laws – Circular Motion

Questions 3 and 4 refer to the following:
A mass is suspended on the end of a string and given a push so that it swings around in a horizontal circle. Because the string sweeps out a cone as the mass goes around, this is called a conical pendulum.



3. A conical pendulum has a length of 1.7 m and a mass of 250 grams. It is spinning such that the tension in the string is 4 N.
   a. What is the net force on the mass?

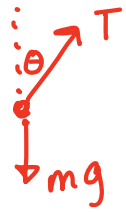
$r = L\sin\theta$

$\Sigma F_y = 0$

$T\cos\theta - mg = 0$

$\cos\theta = \frac{mg}{T} = \frac{(.25)(10)}{4}$

$\theta = 51.3^\circ$



$\Sigma F_x = \frac{mv^2}{r}$

$T\sin\theta = \frac{mv^2}{r}$

So $\Sigma F = T\sin\theta$

$= 4\sin 51.3$

$\Sigma F = 3.12$ N

   b. How fast (m/s) is the mass spinning?

$\Sigma F = \frac{mv^2}{r}$

$3.12 = \frac{(.25)v^2}{1.7\sin 51.3}$

$v^2 = 16.56$

$v = 4.07$ m/s

4. Another conical pendulum has a length of 1.3 m and a mass of 0.6 kg. It is spinning around in a circle such that the angle θ is 30°. How fast is the mass spinning?

$\Sigma F_y = 0$

$T\cos\theta - mg = 0$

$T = \frac{mg}{\cos\theta}$

$\Sigma F_x = \frac{mv^2}{r}$

$T\sin\theta = \frac{mv^2}{L\sin\theta}$

$mg\tan\theta = \frac{mv^2}{L\sin\theta}$

$v^2 = Lg\tan\theta\sin\theta$

$v^2 = (1.3)(10)(\tan 30)(\sin 30)$

$v^2 = 3.75$

$v = 1.94$ m/s

5. A 75 kg person is riding a Ferris Wheel with a 25 meter radius. It is rotating at constant rate of 2 rpm.
   a. What is the magnitude of the net force on the person?

$\left(\frac{2\text{ rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(25)\text{ m}}{1\text{ rev}}\right)$

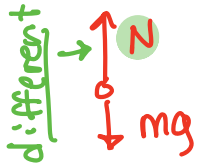
$= 5.24$ m/s

$\Sigma F = \frac{mv^2}{r} = \frac{(75)(5.24)^2}{25}$

$\Sigma F = 82.3$ N

   b. What is the normal force on the person when at the highest point?

Note: these are different!!



@ highest point, center is below person so

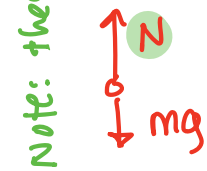
$\Sigma F = \frac{mv^2}{r}$ DOWN → $mg - N = \frac{mv^2}{r}$

$N = mg - \frac{mv^2}{r}$

$N = (75)(10) - 82.3$

$N = 668$ N

   c. What is the normal force on the person when at the lowest point?



@ lowest point, center of circle is above the person, so

$\Sigma F = \frac{mv^2}{r}$ UP → $N - mg = \frac{mv^2}{r}$

$N = mg + \frac{mv^2}{r}$

$N = 750 + 82.3$

$N = 832\,N$

Newton's Laws – Circular Motion

6. A stunt rider on a motorcycle (total mass = 325 kg) is going around a loop-the-loop of radius 15 m. Imagine that they are going around the circle at constant speed of 17 m/s (probably not realistic, but hey it's a physics problem.)

a. What is the normal force of the track on the motorcycle at the very top of the loop?

$\Sigma F = \frac{mv^2}{r}$ DOWN

$N + mg = \frac{mv^2}{r}$

$N = \frac{mv^2}{r} - mg = \frac{(325)(17)^2}{15} - (325)(10)$

$N = 6262 - 3250$

$N = 3012\ N$

b. What is the normal force of the track on the motorcycle at the very bottom of the loop?

$\Sigma F = \frac{mv^2}{r}$ UP

$N - mg = \frac{mv^2}{r}$

$N = mg + \frac{mv^2}{r} = 3250 + 6262$

$N = 9512\ N$

c. What is the minimum speed which the motorcycle can go around the loop and stay on the loop?

Key location is top of loop. Smallest v goes with smallest N. The smallest Normal force is 0, so ...

$\Sigma F = \frac{mv^2}{r}$ ↓

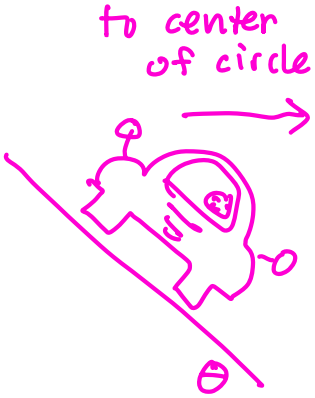
$N + mg = \frac{mv^2}{r}$

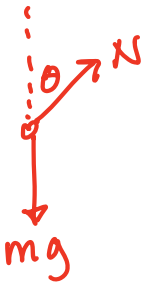
$0 + mg = \frac{mv^2}{r}$

$v^2 = rg = (15)(10)$

$v = 12.3\ m/s$

7. A car is going around a race track with a banked curve. If the car is going 45 m/s, and the radius of the curve is 250 m, what must be the angle of the bank so that friction is not needed to make the curve?





$\Sigma F_x = \frac{mv^2}{r}$ $\Sigma F_y = 0$

$N\sin\theta = \frac{mv^2}{r}$ $N\cos\theta - mg = 0$

$mg\tan\theta = \frac{mv^2}{r}$ ← $N = \frac{mg}{\cos\theta}$

$\tan\theta = \frac{v^2}{rg} = \frac{(45)^2}{(250)(10)}$

$\tan\theta = 0.81$

$\theta = 39^\circ$